A NOVEL METHOD FOR CALCULATION OF LINE VOLTAGE THD IN MULTILEVEL INVERTERS

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Abstract:

Multilevel inverters, the next generation of DC-AC inverters, are widely used in industrial applications especially in high voltage and power. Multilevel inverters include several DC sources. Output voltage waveform of these inverters contains harmonic contents. Total Harmonic Distortion (THD) is the most used index to evaluate harmonic content of any waveform. Calculation of line voltage THD by conventional formula leads to an inexact answer, so a novel method is introduced in this paper to calculate accurate line voltage THD of multilevel inverter. This method can be implemented to each multilevel inverter with any number of higher order levels. To show its superiority, line voltage THD of 3, 17 and 19-level inverters are calculated by proposed method and conventional formula and then, compared to each other. To verify the validity of proposed method, harmonics obtained in approximate method up to 10^6 Order are considered to have an accurate index forcomparison.

Keywords: Accurate THD of line stepped voltage, Multilevel inverter.

INTRODUCTION

Industrial applications such as compensator, switching power supply, electric drive and FACTS device [1] are the practical examples of multilevel inverters [2]-[12]. Their Numerous advantages comparing with conventional inverter (two and three level) lead to these wide applications, especially in high power ones. Harmonic content is one of the most important aspects of these inverters. Output waveform of multilevel inverter usually has staircase form and therefore, introduces harmonic to the network. However, the amount of harmonics, introduced to the network, is less than common inverters because of higher levels of output waveform. Simplest type of multilevel inverter has five levels and the harmonics approach zero by increasing the number of levels. The applicable levels are limited by voltage unbalancing problem, difficulty of voltages control and circuit designrestrictions.

THD is a most used benchmark to measure the harmonic content [13] and it can be utilized to evaluate harmonic generation of multilevel inverters. Root sum square of limited number of harmonics is a common way to calculate THD, but obviously the result is an approximate answer. For instance, [14] has calculated THD up to 49th order, [3] up to 63rd, [15] up to 97th and [16] up to 199thorder but none of them has not been given an accurate THD. Ref. [7] has been proposed a formula to calculate accurate THD of phase voltage of multilevel inverter which can be used for both THD calculation and THD minimization by analytical methods. On the other hand, no method has been proposed yet to calculate line voltage THD of multilevel inverter and study the difference between accurate and approximateTHD.

This paper introduces a novel method to calculate accurate THD of staircase line voltage of multilevel inverter regardless of the number of levels. The validity of proposed method for computing line voltage THD is verified by comparing with the results of approximate method till 10^6 Order. The 3rd, 11th, 15th, 17th and 19th -level inverters are studied here and their line voltage THD is calculated by both proposed and approximate methods in MATLABsoftware.

MULTILEVELINVERTERS

Different configurations of multilevel inverters can be classified as follows: diode-clamped, flying-capacitor and cascaded-inverter with separate DC sources [9]. There are two important switching strategies for determining switching angles; Optimized Harmonic Stepped Waveform (OHSW) [17-18] and Optimal Minimization of Total Harmonic Distortion (OMTHD) [19]. The former is used to remove some low order harmonics and the latter is used to minimize THD totally, without any emphasis on special harmonic. This paper considers the cascade inverter which consists of series H-bridge inverters in each phase. Each H-bridge inverter is called cell, so it can be known as multicell cascade inverter. This inverter synthesizes the desired voltage by several independent DC sources which can be battery, fuel cell or photovoltaic cell. Output waveform is the summation of output voltage of each cell because they are connected in series.

Now, assume a DC source with V_{dc} is connected to one H-bridge inverter with four switches (i.e., S1- S4). Proper switching of S1 to S4 generates +Vdc, -Vdc and zero at the output. The levels of phase voltage is calculated by m = 2s+1, where„*s'* is the number of DC sources (or H-bridges). Fig. 1 shows typical half cycle waveform with switching angles and voltage levels.

This symmetric waveform consists of *S* switching angles in half cycle wave and 4S angles in each cycle (α1, α2, …$\alpha_{(4S-1)}$, $\alpha_{(4S)}$).

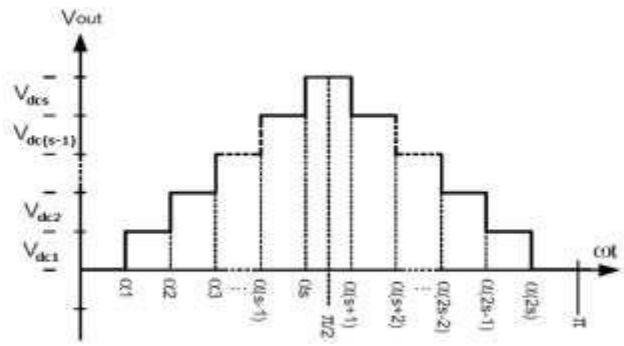


Fig. 1. Waveform of a multicell cascade inverter with *S* cells

Relationships between switching angles of first and other three quarters are as follows:

In second quarter: α_{s+1}=Л-α_s,…, α_{2s-1}= Л-α_2, α_{2s}= Л-α_1
In third quarter:α_{2s+1}=Л+α_1,…, α_{3s-1}=Л+α_{s-1}, α_{3s}= Л+α_s
In fourth quarter:α_{3s+1}=2Л-α_s,…,α_{4s-1}=2Л-α_2,α_{4s}=2Л-α_1

In three-phase system, the output voltage of the three cascaded inverters can be joined in either Wye or Delta connection. In the star topology, maximum number of levels of line voltage is calculated as m = 4s+1 and also, the number of switching angles of line voltage can reach to utmost 2S angles in quarter-wave and consequently, 8S angles in each cycle (α_1, α_2, …, $\alpha_{(8S-1)}$,α_{8S}).

MATHEMATICAL FORMULATION FOR CALCULATION OF LINE VOLTAGE THD

In this part, approximate method to calculate line voltage THD of multilevel inverter is introduced first and then, our proposed method will be explained.

Approximate Method

The conventional formula calculates THD regardless of waveform shape and now, stepped voltage of a general multilevel inverter with unequal DC sources is considered here (Fig. 1). Fourier series of this waveform is given by following:

$$V(\omega t) = \sum_{n=1}^{\infty} Vn * Sin(n\omega t) \quad (1)$$

Where, V_n is the amplitude of voltage harmonics and is given by (2).

$$Vn = \frac{4}{n\pi} \sum_{k=1}^{s} V_{dck} * Cos(n\alpha_k) \quad (2)$$

$$Vn = 0$$

Where, V_{dck} is the voltage of k^{th} DC source. It is obvious that even harmonic orders do not exist in the symmetric waveform. THD of voltage is computed by(3).

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} Vn^2}}{V_1} \quad (3)$$

Where, V_1 is amplitude of fundamental component. Even harmonic orders do not exist in symmetrical waveform and moreover, multiples of 3rd harmonic are eliminated in three phase systems. So, (3) is changed to (4) to calculate line voltage THD of multilevel inverter.

$$THD_{line} = \frac{\sqrt{\sum_{n=5,7,11,13,....}^{\infty} Vn^2}}{V_1} \quad (4)$$

It is seen from (4) that accurate THD is yielded when harmonic orders till infinity are taken into account. But some low orders are always considered in practice that leads to an approximate answer.

Proposed Method

The equivalent form of (3) is given by (5) which can be used to compute THD [10].

$$THD = \sqrt{\left(\frac{V_{rms}}{V_1}\right)^2 - 1} \quad (5)$$

Where, Vrms is Root Mean Square (RMS) of voltage and is given by

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2(\alpha)\, d\alpha} \quad (6)$$

It seems that V_{rms} of a typical line voltage, like Fig. 1, can be calculated easily, but considering Fig. 2 and 3 shows the difficulty of this calculation.

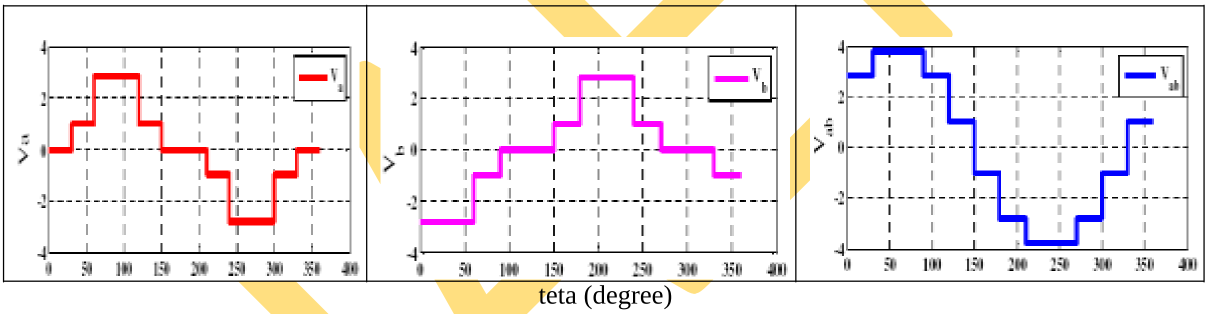


Fig. 2. (a) V_a (b)V_b (c) V_{ab} For a 5-level inverter with α_1=30^0, α_2 =60^0

In both figures, related to a 5-level inverters fired by two different angle sets, line voltage (V_{AB}) has been derived by subtracting phase voltage (V_B) from (V_A) graphically. A remarkable result can be seen; 6-level line voltage waveform is yielded for α_1=30^0, α_2 =60^0 and 9-level for α_1=8^0, α_2 =20^0.

These figures show that there is not a specific waveform for line voltage, so the amount of its V_{rms} cannot be generally calculated because line voltage depends on unknown switching angles of phase B. In other words, switching angles of phase A are specified (α_1, α_2, ..., Л-α_1, Л+α_1, ..., 2Л-α_1) but those of phase B are unknown because first angle of this phase cannot be predicted. For example, the switching angles of line voltage of 5-level inverter cannot be defined generally as a function of two switching angles ($\alpha_1$and α_2). So, inner integralof (6) cannot be calculated generically, unlike phase voltage. Computing (6) is possible when switching angles of phase A have been specified before (e.g. α_1=10^0 and α_2=20° for 5-level inverter). Knowing angles of phase A defines angles of line voltage (V_{AB}) and consequently, V_{rms} can be found by computing inner integral of (6). A novel method to calculate line voltage THD at the base of aforementioned procedure is described, here.

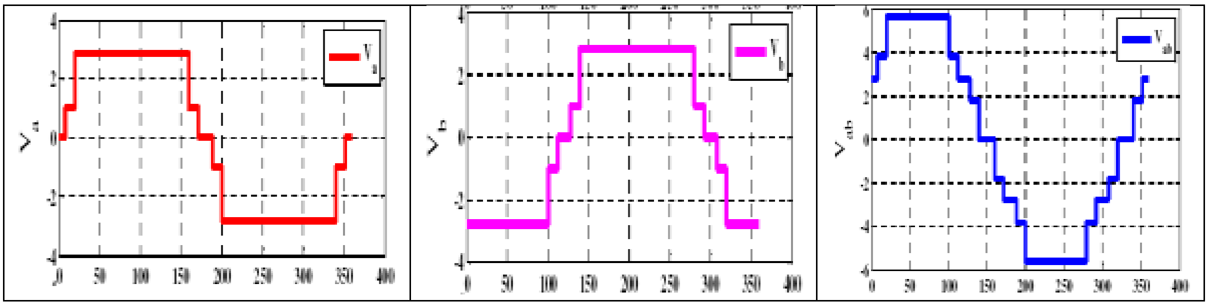


Fig. 3. (a) Va (b)Vb (c)Vab For a 5-level inverter with $\alpha_1=8^0$, $\alpha_2=20^0$

The amount of V_{rms} can be calculated by (6) when a formula is derived for line voltage of multilevel inverter. Considering Fig. 1 clears that this staircase voltage can be defined by step function which is brought as a function of switching angle (α).

$$U(\alpha-\alpha_i) = \begin{cases} 1 & \text{if } \alpha\geq\alpha_i \\ 0 & \text{if } \alpha<\alpha_i \end{cases} \quad (7)$$

Half-cycle of a typical phase voltage, like Fig. 1, is given by (8).

$$P(\alpha) = V_{dc1}U(\alpha-\alpha_1)+\ldots+V_{dcs}U(\alpha-\alpha_s) - V_{dcs}U(\alpha-(\pi-\alpha_s))-\ldots-V_{dc1}U(\alpha-(\pi-\alpha_1)) \quad (8)$$

Second and first half-cycle of symmetric waveforms have horizontal line of symmetry, so phase voltage is totally given by

$$V_a(\alpha) = P(\alpha) - P(\alpha-\pi) \quad (9)$$

V_b has 120° phase shift, comparing with voltage of phase A, which is defined in interval (0, 360°), so V_b is defined by (10).

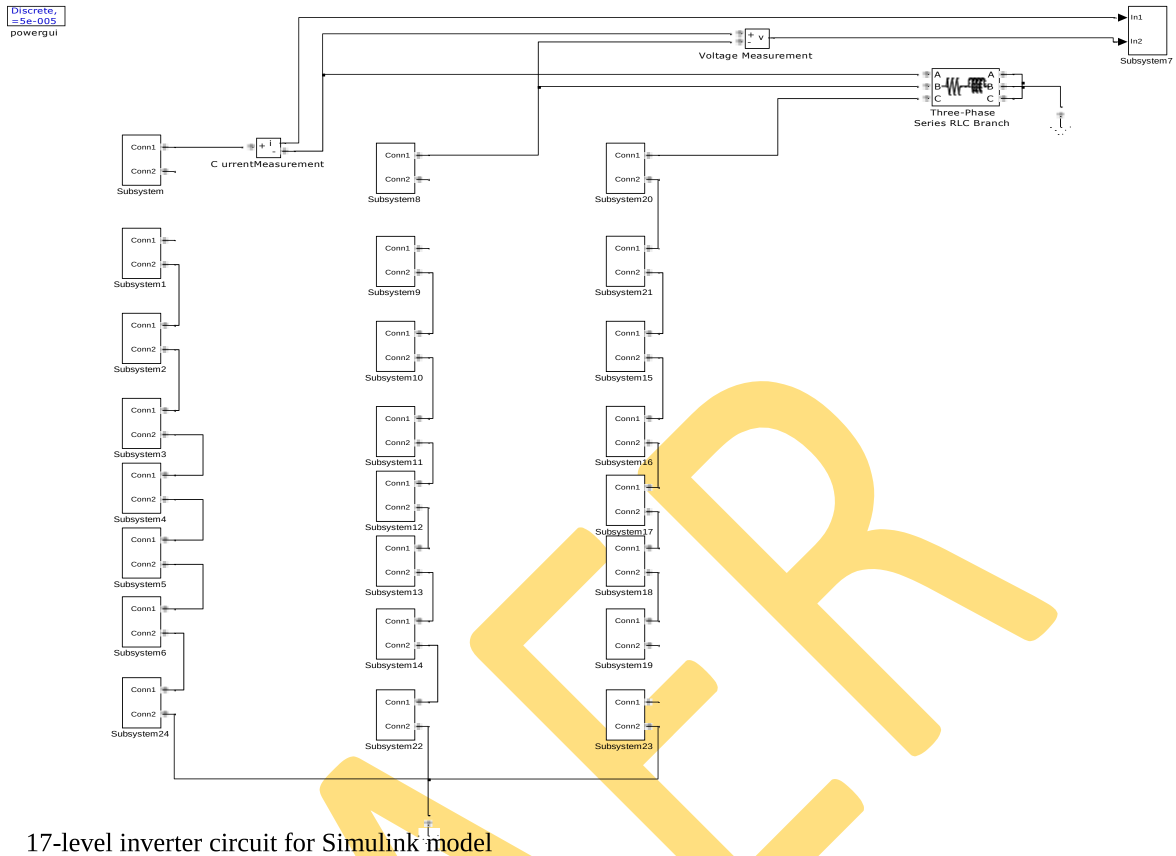
$$V_b(\alpha)=\{V_a(\alpha+120) \quad 0\leq\alpha\leq240 \quad (10)$$

$$V_b(\alpha)=\{V_a(\alpha-120) \quad \text{Otherwise}$$

Finally, line voltage formula versus step function is derived by subtracting $V_b(\alpha)$ from $V_a(\alpha)$. $V_{ab}(\alpha)$ has a constant value in each two neighbor angles ($\alpha_k$and α_{k+1}), so inner integral of (6) can be calculated as follows. The mean of each angle with its neighbor is inserted in $V_{ab}(\alpha)$ and the result is squared. The amount of integral in this interval is calculated by multiplying by (α_{k+1}-α_k). Finally, summation of these results gives the answer of integral and consequently, V_{rms} and line voltage THD are calculated. It must be mentioned that aforementioned formulas cannot be computedparametrically.

PROPOSED SIMULINK MODEL CIRCUIT

The proposed Simulink model circuit for 17-level inverter is shown below.



17-level inverter circuit for Simulink model

SIMULATION RESULTS

Approximate method gives an answer with lesser accuracy; even if large numbers of harmonic orders take into account. Our method gives an answer with ideal accuracy and notable difference with the answer of approximate method. These are shown in this part by simulation in MATLAB.

Our proposed method was introduced in part III and now, the results of it are compared with those of approximate method for different multilevel inverters to verify the validity and accuracy of proposed method. Two approximate line voltage THD based on conventional formula is used in these comparisons; first has been calculated up to 49^{th} harmonic order (THD_{49}) [14] and the other up to 63^{th} order (THD_{63}) [17]. $THD_{accurate}$ is the representative of accurate THD due to proposed method. Fig. 4 shows these values for a 3-level inverter for all accessible angles (0-90°) (line voltage THD versus α). The difference between three THD seems trivial but further attention clears that the reason is large scaling of vertical axis because THD approaches infinity when α approaches 90°.

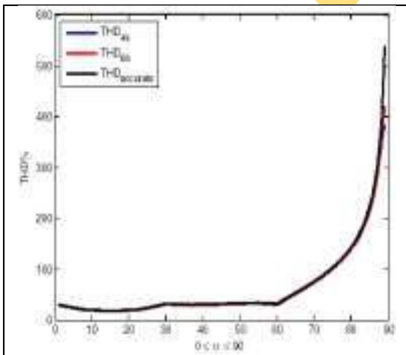
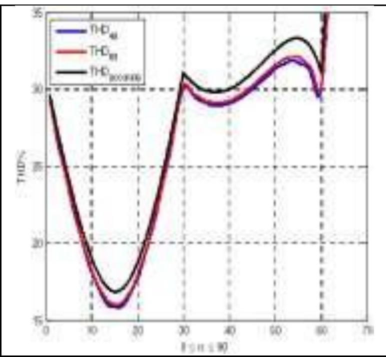
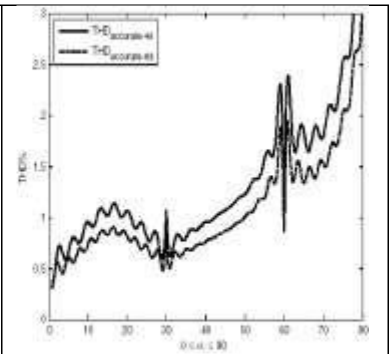
		

Fig. 4. Calculated line voltage THD of 3-levelinverter	Fig. 5. Comparison between calculated line voltage THD	Fig. 6. $THD_{accurate-49}$ and $THD_{accurate-63}$

Fig. 5 which focuses on one part of Fig. 4 clears that line voltage THD due to proposed method is more accurate than others and among them, THD_{49} has least accuracy. It seems that line voltage THD of 3-level inverter has three local minimums and two local maximums in their relevant angles. Also, left and right derivative of THD in 30° and 60° are unequal which illustrates existence of multi criteria function for line voltage THD in each boundary.

Fig. 6 shows difference between THD of accurate method and others in more details. Difference between $THD_{accurate}$ and THD_{49} is called $THD_{accurate-49}$ and $THD_{accurate-63}$ is representative of difference between $THD_{accurate}$ and THD_{63}. Larger amount of $THD_{accurate-49}$ confirms further approximation in the calculation of THD_{49}. According to Fig. 6, the difference between $THD_{accurate}$ and THD_{49} for those angles which are larger than 71°, is more than 2%. Having 2% error in calculation of THD is not acceptable when allowable harmonic of line voltage is 5% regarding to IEEE-519 standard[20].

Proposed method is exerted on 11 and 15-level inverters respectively to show its proficiency in these inverters. Switching angles of these inverters are larger than two angles and cannot be shown in two or three dimensional space, so the results are brought in tables.

$\alpha_1 = 15°$ $\alpha_2 = 25°$ $\alpha_3 = 40°$ $\alpha_4 = 55°$ $\alpha_5 = 60°$		
THD	**Magnitude (%)**	**Error (%)**
THD_{49}	7.1779	9.363
THD_{63}	7.2350	8.642
THD_{10^6}	7.9190	0.005
$THD_{accurate}$	7.9194	0

Table I. Line Voltage THD of 11-Level Inverter

$\alpha_1 = 10°$ $\alpha_2 = 20°$ $\alpha_3 = 35°$ $\alpha_4 = 45°$ $\alpha_5 = 55°$ $\alpha_6 = 60°$ $\alpha_7 = 65°$		
THD	**Magnitude (%)**	**Error (%)**
THD_{49}	4.1870	18.327
THD_{63}	4.2219	17.647
THD_{10^6}	5.1262	0.007
$THD_{accurate}$	5.1266	0

Table II. Line Voltage THD of 15-Level Inverter

Now, proposed method is exerted on 17 and 19-level inverters respectively to show its proficiency in these higher order level inverters. Tables III , IV shows the line voltage THD of 17 and 19-level inverters which are calculated by proposed and approximate methods. The rows of Tables III , IV relate to line voltage THD for harmonics up to 49^{th}, 63^{rd} and 10^{6th} order.

$\alpha_1 = 10°$ $\alpha_2 = 20°$ $\alpha_3 = 35°$ $\alpha_4 = 45°$ $\alpha_5 = 55°$ $\alpha_6 = 60°$ $\alpha_7 = 65°$ $\alpha_8 = 70°$		
THD	**Magnitude (%)**	**Error (%)**
THD_{49}	3.1565	12.0163
THD_{63}	3.0255	15.6678
THD_{10^6}	3.4258	4.5099
$THD_{accurate}$	3.5876	0

Table III. Line Voltage THD of 17-Level Inverter

$\alpha_1 = 10°$ $\alpha_2 = 20°$ $\alpha_3 = 35°$ $\alpha_4 = 45°$ $\alpha_5 = 55°$ $\alpha_6 = 60°$ $\alpha_7 = 65°$ $\alpha_8 = 70°$ $\alpha_9 = 75°$		
THD	**Magnitude (%)**	**Error (%)**
THD_{49}	2.0255	9.1785
THD_{63}	2.0000	10.3219
THD_{10^6}	2.2302	0
$THD_{accurate}$	2.2302	0

Table IV. Line Voltage THD of 19-Level Inverter

CONCLUSION

This paper proposed a novel method to calculate accurate value of line voltage THD of multilevel inverter instead of conventional ones which leads to an approximate answer. This method is a general way to compute line voltage THD of any multilevel inverter. It was shown that the difference between accurate and approximate THD depends on switching angles, but the answer of approximate method usually contains unacceptable approximation. However, the large number of harmonics up to 10^6 order are taken into account can give almost accurate answer, whereas the proposed method, yields an accurate answer and the calculation speed is also faster. Therefore, proposed method should be used as a standard method to calculate the THD of line and phase voltage in multilevel inverters.

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